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Note Additional to a Former Paper "On Certain Ruled Surfaces of the Fourth Order."

By Thomas F. Holgate.

In a paper entitled "On Certain Ruled Surfaces of the Fourth Order," published in volume XV of this Journal, I discussed those surfaces of the fourth order which may be generated by two projectively related sheaves of planes of the second order, or by the reciprocal method of two projectively related conics. It was there shown that when the collinear bundles in which the two projectively related sheaves of planes are chosen have a self-corresponding ray, the surface generated will have as nodal curve two straight lines which may be either real and distinct, coincident, or imaginary, and a double generator, namely, the self-corresponding ray of the bundles. The surface for which the nodal lines are real and distinct, F_5^4, and that for which the nodal lines are coincident, F_6^4, were discussed, but no mention was made of the surface for which the nodal lines are imaginary, though the existence of such a surface must have been in mind at the time.* From the algebraic point of view it would perhaps be unnecessary to distinguish between these three surfaces, but from the geometrical standpoint a study of the separate surfaces is of considerable interest.

Before taking up the study of this surface, however, I wish to call attention to a misstatement in my former paper.

In article 40, page 380, the double generator on the surface F_5^4 is spoken of as being *real* or *imaginary* according as the points O_1 and O_2 in which the double generator meets the nodal lines k_1 and k_2 respectively, lie without or within the generating cones. It should have been stated that the double generator *lies actually on the surface* or *is isolated* according as these points lie without or within the generating cones. In the discussion of the subforms of this surface in article 41, again in article 45, and in the treatment of the surface F_6^4 (art. 47), the same mistake is made and should be similarly corrected.

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*See former paper, article 39, page 379.

Two collinear bundles of planes S and S', which have a self-corresponding ray SS', generate a line congruence of the first order and first class. When the two self-corresponding planes of the bundles, α_1 and α_2, are real and distinct, the singular points of the congruence lie upon two real and distinct straight lines k_1 and k_2, which are gauche to each other and meet the self-corresponding ray SS' in points O_1 and O_2. If these planes are coincident, k_1 and k_2 coincide. If the self-corresponding planes are imaginary, there are no real singular points of the congruence outside the ray SS'.

Suppose that in the latter case two projectively related sheaves of planes of the second order Φ_7 and Φ_7', enveloping cones K_7 and K_7' respectively, be chosen. These will generate a ruled surface of the fourth order, F_7^4, whose generators are rays of the congruence that is generated by S and S'. No two generators of this surface can intersect, since the congruence in which it lies has no real singular points; consequently, the surface has no real nodal lines. Two planes of the sheaf Φ_7 and the two corresponding planes of the sheaf Φ_7' will, in general, pass through the self-corresponding ray SS'. This ray will, therefore, be a double generator of the surface.

If the cones K_7 and K_7' be so chosen that the ray SS' lies outside either of them, and, consequently, also outside the other, the double generator will lie actually upon the surface and the two sheets of the surface through it will be distinct. If SS' lies on the cones, the surface will be torsal all along this generator, while if this self-corresponding ray lies inside the cones, the double generator will be isolated.

The section of the surface F_7^4, made by an arbitrary plane, is a quartic curve having a real double point where the plane intersects the double generator and two imaginary double points. The section by a plane through an arbitrary generator consists of this generator and a cubic curve which has a double point at the intersection of the plane with the double generator and which intersects the generator of its plane in only one real point, namely, the point at which the plane is tangent to the surface. The real double point in both these cases is a crunode a cusp, or an acnode, according as the double generator of the surface is crunodal, torsal, or isolated.

The section of the surface made by a plane through the double generator consists of this line counting doubly and a conic which intersects the line in those points at which the plane is tangent to the surface. These points are real and distinct, coincident, or imaginary, according as the double generator again is crunodal, torsal, or isolated. The planes through the double generator are the only real bitangent planes of the surface.

That the surface F_7^4 is self-reciprocal may be shown by a consideration of the sections made by two bitangent planes, exactly as was done for the surface F_5^4 in the original paper.

For convenience of reference I here append a table showing the principal characteristics of the various forms of Ruled Surfaces of the Fourth Order, including those which do not yield themselves to the projective methods of these papers.

SURFACES WHICH ADMIT A TRINODAL QUARTIC SECTION. $p = 0$.						
				CORRESPONDING SPECIES.		
SPECIES.	Character of the real nodal curve.	Forms constituted by real bitangent planes.	Reciprocal surface.	Cayley.	Cremona.	Salmon.
F_1	A twisted cubic; no straight line director.	A sheaf of the third order.	Of the same form.	10	1	VI.
F_2	A twisted cubic, with a straight line director.	A sheaf of the first order, each plane tritangent.	F_2'	8	7	VII.
F_3	A conic and an intersecting straight line which is not a generator.	A sheaf of the first order and one of the second order.	Of the same form.	7	2	VIII.
F_4	A conic and an intersecting straight line which is also a generator.	A sheaf of the first order, each plane tritangent.	F_4'	11	4	IX.
F_5	Two distinct straight lines and a double generator.	Three distinct sheaves of the first order.	Of the same form.	2	5	X, say.
F_6	One straight line and a double generator.	Two sheaves of the first order.	“	5	6	XIII, say.
F_7	A double generator.	A sheaf of the first order.	“	—	—	—
F_2'	A triple line; no single line director.	A sheaf of the third order.	F_2	9	8	I.

SURFACES WHICH ADMIT A TRINODAL QUARTIC SECTION. $p=0$.

SPECIES.	Character of the real nodal curve.	Forms constituted by real bitangent planes.	Reciprocal surface.	CORRESPONDING SPECIES.		
				Cayley.	Cremona.	Salmon.
F_4	A triple line which is a generator.	A sheaf of the first order and one of the second order.	F_4	12	3	III.
—	A triple line, with a single line director.	A sheaf of the first order, each plane tritangent.	Of the same form.	3	9	II.
—	A triple line; not a generator.	A sheaf of the first order, each plane tritangent.	"	6	10	IV and V.
SURFACES WHICH DO NOT ADMIT A TRINODAL QUARTIC SECTION. $p=1$.						
—	Two distinct straight lines, but no double generator.	Two sheaves of the first order.	Of the same form.	1	11	XI, say.
—	One straight line, but no double generator.	One (doubly counting) sheaf of the first order.	"	4	12	XII, say.

From the above table it will be observed that the two self-reciprocal triple-line surfaces and the two surfaces which do not admit a trinodal quartic section (Cayley's third, sixth, first, and fourth species) have not been treated in these papers; and, further, that neither Cayley, Cremona, nor Salmon makes mention of the surface for which the two nodal lines are imaginary, although they all distinguish the surface for which these lines coincide from that for which the lines are real and distinct.

In a paper in volume XXI of this Journal, Dr. E. M. Blake has described simple mechanisms for generating many of the forms and subforms of the surfaces discussed in these papers, and I should add that it was through correspondence with him that my attention was called to the omissions in my former paper.

NORTHWESTERN UNIVERSITY, EVANSTON, *August* 20, 1898.